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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD COLLEGE.

ON THE CORRECTION FOR THE EFFECT OF THE COUNTER ELECTROMOTIVE FORCE INDUCED IN A MOVING COIL GALVANOMETER WHEN THE INSTRUMENT IS USED BALLISTICALLY.

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At the present time, when on account of their many good qualities ballistic galvanometers of the moving coil type are in such general use, and when some of them fall into the hands of persons who are not acquainted with their vagaries, it seems desirable to emphasize the fact that the counter electromotive force induced in the coil as it swings in the field of its own permanent magnet is often large, and that in certain classes of absolute measurements the result might be in error by many per cent if the effect of this electromotive force were not allowed for. Of course, in comparing condenser charges, mirror, ballistic d'Arsonval galvanometers are usually entirely satisfactory, and, within wide limits, the "throws" of the coil, as measured on the scale of the instrument, are nearly proportional to the quantities of electricity sent impulsively through the coil. If, however, a moving coil galvanometer, the resistance of which is known, be connected in series with a resistance r, wound non-inductively, and an earth inductor, permanently mounted at a given station, it will usually be found that if r be varied and if no account be taken of the electromotive force induced in the galvanometer coil, throws will be obtained from a given movement of the earth inductor, the relative values of which can only be explained by assigning to the resistance of the galvanometer a value much greater than the correct one. The amount of this difference between the apparent resistance of the galvanometer and its real resistance depends, it is clear, upon many things, such as the whole effective area of the turns of the coil, the strength of the field about the coil, the stiffness of the gimp by which the coil is suspended, the

relative resistances of the galvanometer and the rest of the circuit, the moment of inertia of the coil, and the magnitude of the coefficients of the air and electromagnetic damping. Two galvanometers which have the same ballistic sensitiveness to a given discharge from a condenser, and equal resistances, may, of course, act very differently so far as the effect of the induced counter electromotive force is concerned when placed in inductive circuits which outside of themselves are alike. It is useless, therefore, to attempt to predict with any great accuracy the magnitude of the disturbance due to the induced current in any particular instance, but it may be worth while to give briefly the general results of observations in some actual cases, by way of illustration.

(A) A certain d'Arsonval ballistic galvanometer, such that a condenser discharge of one microcoulomb sent through it caused a throw corresponding to a scale reading of about 11.0 cms., was connected in series with a Du Bois and Rubens "Panzer Galvanometer" (P) and with one of two coils of fine insulated wire which formed single layers, the one on a wooden core 2.38 cms. in diameter, the other upon a rod of soft iron 0.94 cms. in diameter. Either coil could be placed within a solenoid of 22.3 turns per centimeter through which a small steady current from a storage cell could be sent, and reversed at pleasure by aid of a commutator in the circuit. By means of a clamp, operated from without the case of the d'Arsonval galvanometer, its coil could be held firmly fixed in such a position that the flux through it of the field of its permanent magnet was practically zero, and it could be released again at will. A heavy suspended system was used in P in order to make the sensitiveness about the same as that of the other instrument when the time of swing was a little shorter than that of the moving coil. The whole resistance of the galvanometer circuit was about 2000 ohms. The slightest movement of the coil of the d'Arsonval galvanometer was sufficient to set the needle of the other instrument in motion also, and the reversal of a given current in the solenoid always caused a larger throw in P when the moving coil was clamped than when it was free. In a typical case the throw of P's needle as measured on the scale of the instrument was 83.5 mm. when the suspended coil was held still, but only 69.9 mm. when it was released. Each of these numbers is the mean of a number of closely accordant readings. Whether the strength of the current in the solenoid was made so small that the throws caused by reversing it could just be read, or so large that they could hardly be measured on the scale, the ratio of the indications when the suspended coil was fixed and when it was free was almost exactly the same; but it was easy to magnify the effect of the electromotive force of the swinging coil by increasing the current in

the solenoid and decreasing the resistance of P so that the throws could be determined. For the purpose of this experiment it was allowable to shunt P, and when a one ninth shunt was used, the "throw" caused by the reversal of a certain current in the solenoid was 73.9 mms. or 45.0 mms. according as the suspended coil was clamped or free.

(B) A certain d'Arsonval ballistic galvanometer the total resistance of which at the room temperature was almost exactly 123.7 B. A. units, was connected in series with a rheostat and an earth inductor of 11.7 B. A. units resistance, the frame of which was fixed at a certain station. The moving coil of the instrument was furnished with a horizontal wire which projected about 23 mms. beyond the coil on each side and carried on each end a brass weight. The object of this device was to increase materially the moment of inertia of the suspended system and thus to make the time of swing larger without increasing the already inconveniently great sensitiveness of the galvanometer. When the inductor ring, which lay in a vertical plane perpendicular to the meridian, was reversed, the galvanometer throw, as measured on the scale at a distance of 120 cms. from the mirror, was 9.41 cms., 5.51 cms., or 3.95 cms., according as the resistance of the rheostat in B. A. units was 0, 100, or 200 : if no account were taken of the counter electromotive force induced in the suspended coil, these figures would point to a circuit resistance of something more than 141 units instead of 135.4 units. In the condition just described the periodic time of the suspended system was 34.5 seconds : when that time was reduced to 12.4 seconds by the removal of the weights attached to the horizontal wire, the throw caused by reversing the inductor ring was 12.98, 7.72, or 5.49 according as the rheostat resistance was 100, 300, or 500 ; these figures point to an apparent resistance of 193 units in the circuit or 42 per cent more than the real resistance.

(C) A certain extremely sensitive d'Arsonval ballistic galvanometer, the coil of which was about twice as long as is usual in such instruments, was connected in series with a rheostat and a "magneto-inductor," i. e., a short coil of very fine insulated wire wound on a spool which could be slipped between near, fixed stops on a long magnetized rod. The permanent magnet of the galvanometer was built up of a large number of thin horseshoe plates hardened separately. The resistance of the galvanometer current when the rheostat resistance was zero was about 8691 at room temperature. When the exploring coil was slipped from one stop to the other, the throw of the galvanometer, as measured on the scale, was 67.0, 58.1, 51.3, 48.3, or 40.05, according as the resistance of the rheostat was 0, 2000, 4000, 5000, or 8500. Of course it is not possible to compute the apparent

resistance of the circuit with any great accuracy from these numbers, any one of which might possibly be in error by 0.5% of its own value and thus introduce a much greater fractional error into the result; but it is interesting to find that if we compare the first number with the others in succession, we get for the apparent resistance of the circuit about 13060, 13070, 12915, 12990, and these numbers point to a value 50% above the real value. The resistance of the "magneto-inductor" alone was 40.3.

(D) When another similar ballistic galvanometer of the same general type was substituted for the one mentioned in (C) so that the resistance of the whole circuit was 4270, Mr. John Coulson obtained throws of 116.0, 100.1, 88.6, 72.5, or 65.8, according as the rheostat resistance was 0, 1000, 2000, 4000, or 5000. If we compare the first of these numbers with each of the others in succession, we get as values of the apparent resistances of the circuit, 6295, 6470, 6665, 6550, and these indicate a value about 50% above the true one.

(E) A d'Arsonval galvanometer such that the resistances of the coil, suspending gimp, and connections amounted in all to only 4.8, was connected in series with a rheostat and the "magneto-inductor" already mentioned. The coefficient of air damping in this galvanometer was considerable, and the field due to the permanent magnet extremely weak. The total resistance of the circuit was 45.1, plus the rheostat resistance. By fastening the stops which limit the journeys of the sliding coil on the inductor rod near together at the centre of the rod or further apart near an end, the electromotive force induced in the sliding coil as it was moved quickly from one stop to the other could be varied within wide limits. Three different positions of the stop were used, and in each case a large number of observations were taken and the mean used, as in all the results here given. For the first position the throw of the galvanometer corresponded to a scale reading of 77.8 mm. or 36.9 mm., according as the rheostat resistance was 0 or 50; these numbers point to a circuit resistance of 45.2, which is almost exactly the true value. The readings for the second position were 129.3 and 61.4, and the calculated resistance was again 45.2. The readings for the third position were 170.4 and 81.7, and the calculated resistance was 46.0 units. The results obtained in this extreme case illustrate what one may expect if the field is extremely weak and the number of turns in the coil small.

(F) Another d'Arsonval galvanometer of very low coil resistance but furnished with much finer gimp so that its total resistance was about 21, and with a strong permanent magnet, was treated in much the same way as the galvanometer mentioned in (E). Two positions of its

stops on the inductor rod were used, and in each of them throws were observed when the rheostat resistance was 0, 50, and 70; the total resistance of the galvanometer circuit outside the rheostat was 60.8. For the first position the throws were comparatively large (196.3, 112.2, and 95.7), and the apparent resistance of the galvanometer circuit was 66.6 besides the rheostat resistance. For the other setting of the stops, the throws were much smaller (42.1, 24.6; 20.95), and the apparent circuit resistance 70.2 ohms. In some galvanometers the apparent resistance, when calculated from large throws as measured on the calibrated scale, will be larger than when deduced from small throws. In other galvanometers the opposite rule holds.

(G) A galvanometer of the same design as the one mentioned in (F), but with many more turns in the coil and a resistance of 730 units, was tested with the "magneto-inductor." This fine instrument, which is furnished with a strong permanent magnet, has been used in making a great number of measurements of permeability, and the very great difference between the apparent and the real resistance (770.3) of the circuit outside of the rheostat, illustrates strikingly the danger of neglecting the correction for the counter electromotive force induced in the coil during a swing. For one setting of the stops on the inductor coil, the readings caused by the galvanometer throw were 66.2 or 39.7, according as the rheostat resistance was 0 or 1000; these numbers point to a circuit resistance of 1498 units. For the second setting of the stops, the readings were 88.2 and 52.9, and these correspond to an apparent circuit resistance of 1499 units. The apparent resistance is in this case nearly double the real resistance. For wide ranges of throws the apparent resistance of any circuit in which this galvanometer may be placed remains almost exactly constant, and the instrument is wholly trustworthy when absolute measurements are to be made, if the apparent resistance instead of the true resistance be used. The apparent resistance of this galvanometer is naturally not quite the same in all circuits; when the electromotive force to be induced in a coil attached to it is larger, it is of course necessary to introduce a large extra resistance into the circuit, and this seems to reduce very slightly the difference between the apparent and the real resistance of the circuit; but so far as my experience goes, it has always been about 700 units. A condenser discharge of one microcoulomb sent through this galvanometer gives a scale reading of about 100 mm.

(H) A very useful instrument for hysteresis work on iron cores of electro-magnets is the microamperemeter of Paul, which has the form of an ordinary portable commercial amperemeter, and is of moderate cost.

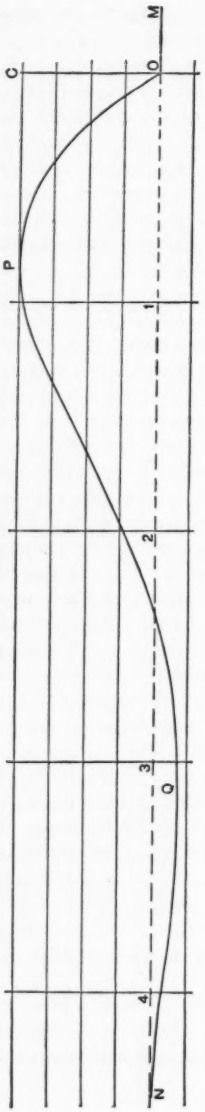

FIGURE 1.

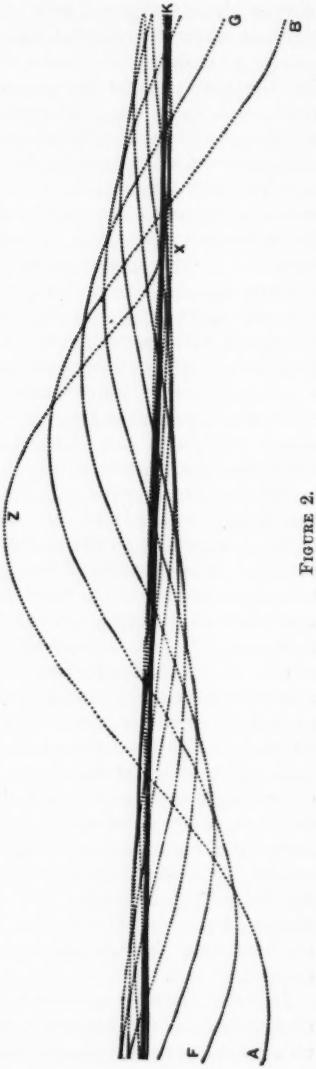

FIGURE 2.

The coil hangs from a single jewel, and the galvanometer is much more sensitive than the "millivoltmeters" usually found in American laboratories. The calibration curve for ballistic purposes is straight and the scale practically one of equal parts; one of these instruments which I have used has a resistance of about 60 ohms, and one microcoulomb sent impulsively through the coil causes a throw of 2 scale divisions. This galvanometer was connected with a rheostat and the low resistance secondary of a small induction coil, the primary of which could be opened and shut by means of a commutator; a rheostat in this primary circuit made it possible to change at pleasure the amount of electricity set in motion in the secondary circuit. The real resistance of the galvanometer circuit was about 74.9, and the apparent resistance for all discharges up to 20 microcoulombs seemed to be somewhere about 147. I took seventeen sets of observations with different quantities, and got some results as low as 143.5 and others as high as 151. The results all lay on a curious wavy line which seemed to mark some idiosyncracy of this particular instrument.

(I) The galvanometers used in (A) to (G) were all of the compact coil form without the inside soft iron core of the original d'Arsonval instrument. Galvanometers of this latter type, however, have their own advantages, as is well known, and are of course in very common use in commercial amperemeters and voltmeters and in laboratory mirror instruments of moderate sensitiveness for use in bridge work and in measuring ballistically condenser charges. As an extremely satisfactory representative of this class I chose one of the Leeds and Northrup Company's "Type H Galvanometers," of which there are several in the Jefferson Laboratory. The calibration curve of this instrument for throws due to condenser discharges is nearly straight, for quantities up to about 4 microcoulombs; the throw due to 1 microcoulomb being about 3.2 cm., whatever the resistance, up to 240,000 ohms, in the discharge circuit. A battery the electromotive force of which was about 8 volts was made to charge — through a resistance large enough (2000 ohms) to prevent possible polarization — a set of standard condensers, which were afterwards discharged through the galvanometer, and the throws corresponding to capacities of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.00 microfarads, were about 2.75, 5.49, 8.21, 10.99, 13.82, 16.71, 19.70, 22.80, 26.02, 29.37. The "magneto-inductor" and rheostat were attached to this galvanometer, and, with the stops on the inductor rod in certain fixed positions, the throws of the moving coil as measured on the scale were 15.81, 13.33, 9.76, 8.19, or 7.05, according as the rheostat resistance was 0, 300, 1000, 1500, or 2000. The first of these numbers in connection with each of the

others gives the four values 1613, 1613, 1612, 1610 for the apparent resistance of the galvanometer circuit. These results, which agree wonderfully with each other considering the slight uncertainty in the readings, differ widely from the true resistance of the circuit, which was 531. If, when the rheostat resistance was zero, the galvanometer coil had been held still, the whole quantity of electricity set in motion in the circuit when the inductor coil was moved between its stops would have been more than three times the actual flux (rather less than 5 microcoulombs) when the coil was allowed to swing. This fact illustrates the magnitude of the error which might be made in a permeability measurement if the true resistance of the circuit instead of the apparent resistance were used. As is evident from the numbers given above, the apparent resistance is nearly independent of the actual resistance in the circuit when the impulse is constant and comes from a given source, but when the character of the impulse is changed the apparent resistance changes slightly also sometimes, and in a manner to be determined for each instrument by experiment. For a certain impulse which caused a throw of 8.52 cm., as indicated on the scale of the galvanometer just mentioned, the apparent resistance of the circuit was about 1704.

In such ballistic magnetic measurements as allow more than one observation at each stage of the experiment, it is possible, by alternately introducing and taking out of the circuit a known resistance of suitable value wound non-inductively, to determine the apparent resistance of the galvanometer circuit for each impulse. In other cases it is feasible to put into the circuit once for all such a large resistance, say 200,000 ohms, that changes in the apparent resistance of the circuit and changes in the inductive impulses become relatively unimportant. So far as my experience goes, a given instrument, as was to be expected, holds its apparent resistance for a given impulse unchanged for an indefinite time.

If a condenser discharge be sent through a d'Arsonval galvanometer which has no electromagnetic damping other than that which accompanies currents in its own coil, the form of the curve which shows the position of the swinging coil as a function of the time is hardly affected at all by the introduction of a large inductance or a resistance of several thousand ohms into the discharging circuit; but if the condenser be displaced by a magneto-inductor of small resistance, the damping is much increased and the shape of the curve is much altered. Figure 1, in which the horizontal divisions represent seconds, shows a careful copy (M O P Q N) on a small scale of a large photographic record of the excursion of the coil of a certain d'Arsonval galvanometer

of short period, when a very sudden impulse from a magneto-inductor was sent through it; the curve starts very abruptly at O. Figure 2 is a photographic reduction, on the same scale as that of the other figure, of the record, just as it was removed from the drum, of an excursion of the same galvanometer when a condenser discharge was sent through it. The straight line through K represents the zero position of the coil; the curve begins at X, but not so abruptly as before. When the paper was on the drum, the points A, B and the points F, G fell together. It will be noticed that the time of the first swing of the coil on one side of its zero is nearly the same, about 2.34 seconds, in both diagrams.

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